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$\Sigma b_i{}^r = k'b_1{}^r + \delta' k' b_1{}^r$, where δ and δ' can be made as small in absolute value as we please by taking r sufficiently large. Since the left-hand sides of these equations are equal, the right-hand sides must be equal also. Equating them we obtain

$$\frac{k}{k'} = \frac{b_1{}^r}{a_1{}^r}\frac{(1+\delta')}{(1+\delta)} = \frac{b_1{}^r}{a_1{}^r}(1+n),$$

where n is an infinitesimal. If, now, a_1 and b_1 are not equal we can make the right-hand side as small (or as large) as we please by taking r sufficiently large. This is impossible for the ratio $k \div k'$ is a definite number. Consequently $a_1 = b_1$, and $k = k'$. In each equation we may now subtract from each side the a's and b's proven equal and proceed as before to prove the largest remaining a's and b's to be equal in value and number. We may continue in this way until there are no a's (or b's) left, when the b's (a's) must also be exhausted.

471 was also solved by C. F. GUMMER and FRANK IRWIN.

473. Proposed by J. I. GINSBURG, Student, Cooper Union, New York.

Factor the expression $x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1$.

SOLUTION BY CLARIBEL KENDALL, University of Colorado.

Multiplying the given expression by $x^5 - 1$, we obtain $x^{35} - 1$. This may be factored in the following way:

First, consider

$$\begin{aligned} x^{35} - 1 &= (x^5)^7 - 1 = (x^5 - 1)(x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1) \\ &= (x - 1)(x^4 + x^3 + x^2 + x + 1)(x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1). \end{aligned} \tag{1}$$

Next consider

$$\begin{aligned} x^{35} - 1 &= (x^7)^5 - 1 = (x^7 - 1)(x^{28} + x^{21} + x^{14} + x^7 + 1) \\ &= (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)(x^{28} + x^{21} + x^{14} + x^7 + 1). \end{aligned} \tag{2}$$

The factors of $x^{35} - 1$ must be the same in both (1) and (2). $x - 1$ occurs in both. The prime factor $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ in (2) must also be found in (1). It cannot be contained in the prime factor $x^4 + x^3 + x^2 + x + 1$ and hence must be found in the third factor. By division the factors of

$$x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1$$

are found to be

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

and

$$x^{24} - x^{23} + x^{19} - x^{18} + x^{17} - x^{16} + x^{14} - x^{13} + x^{12} - x^{11} + x^{10} - x^8 + x^7 - x^6 + x^5 - x + 1.$$

Also variously solved by H. C. FEEMSTER, HORACE OLSON, O. S. ADAMS, W. F. SHELTON, J. W. BALDWIN, A. W. SMITH, E. B. ESCOTT, L. C. MATHEWSON, PAUL CAPRON, E. J. OGLESBY, and the PROPOSER.

GEOMETRY.

493. Proposed by FLORENCE P. LEWIS, Goucher College, Baltimore, Md.

Construct three circles each of which shall be tangent to the other two and to two sides of a given triangle.

NOTE BY J. L. COOLIDGE, Harvard University.

It is always a pleasure to see this old friend. Few problems in elementary geometry have a longer or more distinguished pedigree. At least forty articles dealing with it were published in the nineteenth century and the twentieth is doing its share.[1] The history is briefly this. The

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[1] SIMON, *Die Entwickelung der Elementargeometrie im XIX Jahrhundert*, Leipzig, 1906, pp. 147ff.